Chapter 26. Trigonometrical Ratios

Ex 26.1

Answer 1.

(i) $\text{Sin}A = \frac{12}{13}$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{12}{13}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144} = \sqrt{25}$$
$$= 5$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\sec A = \frac{1}{\cos A} = \frac{13}{5}$$

$$\cot A = \frac{1}{\tan A} = \frac{5}{12}$$

$$\text{cosecA} = \frac{1}{\sin A} = \frac{13}{12}$$

(ii) $\cos B = \frac{4}{5}$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(5)^2 - (4)^2} = \sqrt{25 - 16} = \sqrt{9}$$

$$= 3$$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

$$\sec B = \frac{1}{\cos B} = \frac{5}{4}$$

$$\cot B = \frac{1}{\tan B} = \frac{4}{3}$$

$$\text{cosecB} = \frac{1}{\sin B} = \frac{5}{3}$$

(iii) $\cot A = \frac{1}{11}$

$$\cot A = \frac{1}{\tan A} = \frac{\text{Base}}{\text{Perpendicular}}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$(\text{Hypotenuse}) = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$

$$= \sqrt{(11)^2 + (1)^2} = \sqrt{121 + 1} = \sqrt{122}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{\sqrt{122}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = 11$$

$$\sec A = \frac{1}{\cos A} = \sqrt{122}$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{11}{\sqrt{122}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sqrt{122}}{11}$$

(iv) $\operatorname{cosec} C = \frac{15}{11}$

$$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{15}{11}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(15)^2 - (11)^2} = \sqrt{225-121} = \sqrt{104}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{11}{15}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{104}}{15}$$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{11}{\sqrt{104}}$$

$$\sec C = \frac{1}{\cos C} = \frac{15}{\sqrt{104}}$$

$$\cot C = \frac{1}{\tan A} = \frac{\sqrt{104}}{11}$$

(iv) $\operatorname{cosec} C = \frac{15}{11}$

$$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{15}{11}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(15)^2 - (11)^2} = \sqrt{225-121} = \sqrt{104}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{11}{15}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{104}}{15}$$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{11}{\sqrt{104}}$$

$$\sec C = \frac{1}{\cos C} = \frac{15}{\sqrt{104}}$$

$$\cot C = \frac{1}{\tan A} = \frac{\sqrt{104}}{11}$$

(v) $\tan C = \frac{5}{12}$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{5}{12}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$(\text{Hypotenuse}) = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$

$$= \sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169}$$
$$= 13$$

$$\cot C = \frac{1}{\tan C} = \frac{12}{5}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{12}{13}$$

$$\sec C = \frac{1}{\cos C} = \frac{13}{12}$$

$$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{13}{5}$$

(vi) $\sin B = \frac{\sqrt{3}}{2}$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(2)^2 - (\sqrt{3})^2} = \sqrt{4-3} = \sqrt{1}$$

$$= 1$$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{2}$$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \sqrt{3}$$

$$\sec B = \frac{1}{\cos B} = 2$$

$$\cot B = \frac{1}{\tan B} = \frac{1}{\sqrt{3}}$$

$$\text{cosec} B = \frac{1}{\sin A} = \frac{2}{\sqrt{3}}$$

(vii) $\cos A = \frac{7}{25}$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{7}{25}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(25)^2 - (7)^2} = \sqrt{625-49} = \sqrt{576}$$

$$= 24$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{24}{25}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{24}{7}$$

$$\sec A = \frac{1}{\cos A} = \frac{25}{7}$$

$$\cot A = \frac{1}{\tan A} = \frac{7}{24}$$

$$\text{cosec} A = \frac{1}{\sin A} = \frac{25}{24}$$

(viii) $\tan B = \frac{8}{15}$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{8}{15}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$(\text{Hypotenuse}) = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$
$$= \sqrt{(8)^2 + (15)^2} = \sqrt{64 + 225} = \sqrt{289}$$
$$= 17$$

$$\cot B = \frac{1}{\tan B} = \frac{15}{8}$$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{8}{17}$$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\sec B = \frac{1}{\cos B} = \frac{17}{15}$$

$$\text{cosec} B = \frac{1}{\sin B} = \frac{17}{8}$$

(ix) $\sec B = \frac{15}{12}$

$$\sec B = \frac{1}{\cos B} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{15}{12}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$
$$\Rightarrow \text{Perpendicular} = \sqrt{(15)^2 - (12)^2} = \sqrt{225 - 144} = \sqrt{81}$$
$$= 9$$

$$\sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{9}{15}$$

$$\tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{9}{12}$$

$$\cot B = \frac{1}{\tan B} = \frac{12}{9}$$

$$\text{cosec} B = \frac{1}{\sin B} = \frac{15}{9}$$

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{12}{15}$$

(x) $\operatorname{cosec} C = \sqrt{10}$

$$\operatorname{cosec} C = \frac{1}{\sin C} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$$

By Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow \text{Base} = \sqrt{(\sqrt{10})^2 - (1)^2} = \sqrt{10-1} = \sqrt{9}$$
$$= 3$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{\sqrt{10}}$$

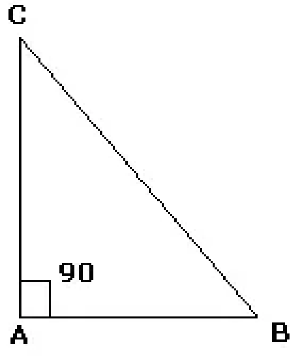
$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{\sqrt{10}}$$

$$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{3}$$

$$\sec C = \frac{1}{\cos C} = \frac{\sqrt{10}}{3}$$

$$\cot C = \frac{1}{\tan A} = 3$$

Answer 2.



In ΔABC,

$$BC^2 = AB^2 + AC^2$$
$$\Rightarrow BC = \sqrt{AB^2 + AC^2}$$
$$\Rightarrow BC = \sqrt{5^2 + 12^2}$$
$$= \sqrt{169} = 13$$

$AC = 12 \text{units}$

$BC = 13 \text{units}$

$AB = 5 \text{units}$

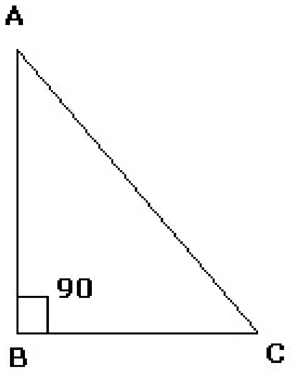
$$\text{(i)} \sin B = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AC}{BC} = \frac{12}{13}$$

$$\text{and} \quad \cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{BC} = \frac{5}{13}$$

$$\text{(ii)} \cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AC}{BC} = \frac{12}{13}$$

$$\text{(iii)} \tan B = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AC}{AB} = \frac{12}{5}$$

Answer 3.



In ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC = \sqrt{AB^2 + BC^2}$$

$$\Rightarrow AC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= 13$$

$$AB = 12 \text{units}$$

$$BC = 5 \text{units}$$

$$AC = 13 \text{units}$$

$$\text{(i) } \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$\text{(ii) } \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{5}{12}$$

$$\text{(iii) } \cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$\text{(iv) } \cot C = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB} = \frac{5}{12}$$

Answer 4.

$$\sin A = \frac{3}{5} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

By Pythagoras theorem, we have

$$\Rightarrow (\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow (\text{Base})^2 = (\text{Hypotenuse})^2 - (\text{Perpendicular})^2$$

$$\Rightarrow (\text{Base}) = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$\Rightarrow (\text{Base}) = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

Answer 5.

$$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$(AB)^2 = (AC)^2 + (BC)^2$$

$$\Rightarrow AC = \sqrt{(AB)^2 - (BC)^2}$$

$$\Rightarrow AC = \sqrt{3^2 - 1} = \sqrt{9-1} = 2\sqrt{2}$$

$$\sin A = \frac{BC}{AB} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{3}$$

$$\tan B = \frac{AC}{BC} = \frac{\text{Perpendicular}}{\text{Base}} = 2\sqrt{2}$$

$$\cot A = \frac{1}{\tan A} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AC}{BC} = 2\sqrt{2}$$

Answer 6.

$$\sin\theta = \frac{8}{17} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$= \sqrt{17^2 - 8^2} = \sqrt{225} = 15$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{8}{15}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{17}{8}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{17}{15}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{15}{8}$$

Answer 7.

$$\tan A = 0.75 = \frac{75}{100} = \frac{3}{4} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$
$$= \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$$
$$= 5$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5} = 0.6$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5} = 0.8$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{3} = 1.66$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{4} = 1.25$$

$$\cot A = \frac{1}{\tan A} = \frac{4}{3} = 1.33$$

Answer 8.

$$\sin A = 0.8 = \frac{8}{10} = \frac{4}{5} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$
$$= \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{5} = 0.6$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{4}{3} = 1.33$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4} = 1.25$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{3} = 1.66$$

$$\cot A = \frac{1}{\tan A} = \frac{3}{4} = 0.75$$

Answer 9.

$$8\tan\theta = 15$$

$$\Rightarrow \tan\theta = \frac{15}{8} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$

$$= \sqrt{15^2 + 8^2}$$

$$= \sqrt{225 + 64} = \sqrt{289}$$

$$= 17$$

$$\text{(i)} \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\text{(ii)} \cot\theta = \frac{1}{\tan\theta} = \frac{8}{15}$$

$$\text{(iii)} \sin^2\theta - \cot^2\theta = (\sin\theta + \cot\theta)(\sin\theta - \cot\theta)$$

$$= \left(\frac{15}{17} + \frac{8}{15}\right)\left(\frac{15}{17} - \frac{8}{15}\right)$$

$$= \left(\frac{225 + 136}{255}\right)\left(\frac{225 - 136}{255}\right)$$

$$= \left(\frac{361}{255}\right)\left(\frac{89}{255}\right) = \frac{32129}{65025}$$

Answer 14.

We are given that $BD : DC = 1 : 2$ as AD divides BC in the ratio $1 : 2$.

i.e $BD = x, DC = 2x \Rightarrow BC = 3x$

$$\text{(i)} \frac{\tan\angle BAC}{\tan\angle BAD} = \frac{\frac{BC}{AB}}{\frac{BD}{AB}} = \frac{BC}{BD} = \frac{3x}{x} = 3$$

$$\text{(ii)} \frac{\cot\angle BAC}{\cot\angle BAD} = \frac{\frac{AB}{BC}}{\frac{AB}{BD}} = \frac{BD}{BC} = \frac{x}{3x} = \frac{1}{3}$$

Answer 19.

As PS is the median on QR from P.

$\therefore QS = SR \Rightarrow QR = 2QS$

and RT divides PQ in the ratio 1 : 2

$\therefore QT = x$ and $PT = 2x$

$\Rightarrow PQ = 3x$

(i) $\dfrac{\tan\angle PSQ}{\tan\angle PRQ} = \dfrac{\frac{PQ}{QS}}{\frac{PQ}{QR}} = \dfrac{PQ}{QS} \times \dfrac{QR}{PQ} = \dfrac{2QS}{QS} = 2$

(ii) $\dfrac{\tan\angle TSQ}{\tan\angle PRQ} = \dfrac{\frac{QT}{QS}}{\frac{PQ}{QR}} = \dfrac{QT}{QS} \times \dfrac{QR}{PQ} = \dfrac{x}{QS} \times \dfrac{2QS}{3x} = \dfrac{2}{3}$

Answer 22.

$24\cos\theta = 7\sin\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{24}{7}$

$\Rightarrow \tan\theta = \dfrac{24}{7} = \dfrac{\text{Perpendicular}}{\text{Base}}$

$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$

$= \sqrt{(24)^2 + (7)^2}$

$= \sqrt{576 + 49} = \sqrt{625} = 25$

$\sin\theta + \cos\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} + \dfrac{\text{Base}}{\text{Hypotenuse}}$

$= \dfrac{24}{25} + \dfrac{7}{25} = \dfrac{24 + 7}{25} = \dfrac{31}{25}$

Answer 24.

$8\tan A = 15$

$\Rightarrow \tan A = \dfrac{15}{8} = \dfrac{\text{Perpendicular}}{\text{Base}}$

$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$

$= \sqrt{(15)^2 + (8)^2}$

$= \sqrt{225 + 64} = \sqrt{289} = 17$

$\sin A - \cos A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} - \dfrac{\text{Base}}{\text{Hypotenuse}}$

$= \dfrac{15}{17} - \dfrac{8}{17} = \dfrac{15 - 8}{17}$

$\sin A - \cos A = \dfrac{7}{17}$

Answer 25.

$$3\cos\theta - 4\sin\theta = 2\cos\theta + \sin\theta$$
$$\Rightarrow 3\cos\theta - 2\cos\theta = \sin\theta + 4\sin\theta$$
$$\Rightarrow \cos\theta = 5\sin\theta$$
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{5}$$
$$\Rightarrow \tan\theta = \frac{1}{5}$$

Answer 26.

$$\text{If } 5\cos\theta = 3$$
$$\Rightarrow \cos\theta = \frac{3}{5} = \frac{\text{Base}}{\text{Hypotenuse}}$$
$$\text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$
$$= \sqrt{(5)^2 - (3)^2}$$
$$= \sqrt{25 - 9} = \sqrt{16}$$
$$= 4$$
$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{4}{5}$$
$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{4 \times \frac{3}{5} - \frac{4}{5}}{2 \times \frac{3}{5} + \frac{4}{5}} = \frac{\frac{12}{5} - \frac{4}{5}}{\frac{6}{5} + \frac{4}{5}} = \frac{\frac{8}{5}}{\frac{10}{5}} = \frac{4}{5}$$

Answer 28.

$$5\tan\theta = 12$$
$$\Rightarrow \tan\theta = \frac{12}{5} = \frac{\text{Perpendicular}}{\text{Base}}$$
$$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$
$$= \sqrt{(12)^2 + (5)^2}$$
$$= \sqrt{144 + 25} = \sqrt{169} = 13$$
$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{12}{13}, \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13}$$
$$\Rightarrow \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}} = \frac{24 - 15}{48 - 45} = \frac{9}{3} = 3$$

Answer 30.

$$\cot\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}} = \frac{\text{Base}}{\text{Perpendicular}}$$

$$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$

$$= \sqrt{\left(\sqrt{3}\right)^2 + 1} = \sqrt{3+1} = 2$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{2},$$

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\text{To show : } \frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$$

$$\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{1-(\cos\theta)^2}{2-(\sin\theta)^2} = \frac{1-\frac{1}{4}}{2-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$

Answer 31.

$$\operatorname{cosec}\theta = 1\frac{9}{20} = \frac{29}{20}$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{20}{29} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$= \sqrt{(29)^2 - (20)^2} = \sqrt{841-400}$$

$$= \sqrt{441} = 21$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{21}{29}$$

$$\text{To show: } \frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$$

$$\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{1-\frac{20}{29}+\frac{21}{29}}{1+\frac{20}{29}+\frac{21}{29}}$$

$$= \frac{29-20+21}{29+20+21}$$

$$= \frac{30}{70} = \frac{3}{7}$$

Answer 32.

$$b\tan\theta = a$$
$$\Rightarrow \tan\theta = \frac{a}{b}$$

Consider $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

Dividing the numerator and denominator by $\cos\theta$, we get

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1 + \frac{\sin\theta}{\cos\theta}}{1 - \frac{\sin\theta}{\cos\theta}} = \frac{1 + \tan\theta}{1 - \tan\theta}$$

$$= \frac{1 + \frac{a}{b}}{1 - \frac{a}{b}} = \frac{\frac{b+a}{b}}{\frac{b-a}{b}} = \frac{(b+a)}{(b-a)}$$

Answer 33.

$$a\cot\theta = b$$
$$\Rightarrow \cot\theta = \frac{b}{a}$$
$$\Rightarrow \tan\theta = \frac{1}{\cot\theta} = \frac{a}{b}$$

To prove: $\dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \dfrac{a^2 - b^2}{a^2 + b^2}$

Consider $\dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

Dividing the numerator and denominator by $\cos\theta$, we get

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta} - b}{a\frac{\sin\theta}{\cos\theta} + b} = \frac{a\tan\theta - b}{a\tan\theta + b}$$

$$= \frac{a \times \frac{a}{b} - b}{a \times \frac{a}{b} + b} = \frac{\frac{a^2 - b^2}{b}}{\frac{a^2 + b^2}{b}} = \frac{a^2 - b^2}{a^2 - b^2}$$

Answer 34.

$\cot\theta = \sqrt{7}$

$\Rightarrow \frac{\cos\theta}{\sin\theta} = \sqrt{7}$

$\Rightarrow \frac{\text{base}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\sqrt{7}}{1}$

$\Rightarrow \frac{\text{base}}{\text{perpendicular}} = \frac{\sqrt{7}}{1}$

$\text{Hypotenuse} = \sqrt{(\text{perpendicular})^2 + (\text{Base})^2}$

$= \sqrt{1+7} = 2\sqrt{2}$

$\text{To show}: \frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{\left(\frac{\text{hypotenuse}}{\text{perpendicular}}\right)^2 - \left(\frac{\text{hypotenuse}}{\text{base}}\right)^2}{\left(\frac{\text{hypotenuse}}{\text{perpendicular}}\right)^2 + \left(\frac{\text{hypotenuse}}{\text{base}}\right)^2}$$

$$= \frac{\left(\frac{2\sqrt{2}}{1}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(\frac{2\sqrt{2}}{1}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2} = \frac{\frac{8}{1} - \frac{8}{7}}{\frac{8}{1} + \frac{8}{7}} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}} = \frac{48}{64}$$

$$= \frac{3}{4}$$

Answer 35.

$12\,\text{cosec}\theta = 13$

$\Rightarrow \text{cosec}\theta = \frac{13}{12}$

$\Rightarrow \sin\theta = \frac{12}{13} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\Rightarrow \text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})}$

$= \sqrt{(13)^2 - (12)^2} = \sqrt{169-144} = \sqrt{25} = 5$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13}$

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{12}{5}$

$\text{Now, } \frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta}$

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144}{169} - \frac{25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$= \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$$

Answer 36.

$$\cot\theta = \frac{13}{12}$$
$$\Rightarrow \frac{\cos\theta}{\sin\theta} = \frac{13}{12}$$
$$\Rightarrow \frac{\text{base}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{13}{12}$$
$$\Rightarrow \frac{\text{base}}{\text{perpendicular}} = \frac{13}{12}$$
$$\text{Hypotenuse} = \sqrt{(\text{perpendicular})^2 + (\text{Base})^2}$$
$$= \sqrt{(12)^2 + (13)^2} = \sqrt{144 + 169} = \sqrt{313}$$
$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2 \times \frac{12}{\sqrt{313}} \times \frac{13}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2} = \frac{\frac{312}{313}}{\frac{169}{313} - \frac{144}{313}} = \frac{\frac{312}{313}}{\frac{25}{313}}$$
$$= \frac{312}{25}$$

Answer 37.

$$\sec A = \frac{5}{4}$$
$$\Rightarrow \cos A = \frac{4}{5} = \frac{\text{Base}}{\text{Hypotenuse}}$$
$$\text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$
$$= \sqrt{25 - 16} = \sqrt{9} = 3$$
$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$$
$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$
$$\text{To show}: \frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$
$$\text{L.H.S} = \frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)} = \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$$

$$= \frac{\frac{225-108}{125}}{\frac{256-300}{125}} = \frac{117}{-44}$$

$$R.H.S = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A} = \frac{3\left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^3}{1-3\left(\frac{3}{4}\right)^2} = \frac{\frac{9}{4}-\frac{27}{64}}{1-\frac{27}{16}} = \frac{\frac{144-27}{64}}{\frac{16-27}{16}}$$

$$= \frac{117}{64} \times \frac{16}{-11} = \frac{117}{4} \times \frac{1}{-11} = \frac{-117}{44}$$

$$\Rightarrow L.H.S = R.H.S$$

Answer 38.

$$\sin\theta = \frac{3}{4}$$

$$\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{4}$$

$$\text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicular})^2}$$

$$= \sqrt{16-9} = \sqrt{7}$$

$$\text{cosec}\theta = \frac{4}{3}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\sqrt{7}}{3}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{4}{\sqrt{7}}$$

To prove $\sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$

$$\sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{\left(\frac{4}{3}\right)^2 - \left(\frac{\sqrt{7}}{3}\right)^2}{\left(\frac{4}{\sqrt{7}}\right)^2 - 1}}$$

$$= \sqrt{\frac{\frac{16}{9}-\frac{7}{9}}{\frac{16}{7}-1}} = \sqrt{\frac{\frac{16-7}{9}}{\frac{16-7}{7}}} = \sqrt{\frac{\frac{9}{9}}{\frac{9}{7}}}$$

$$= \sqrt{\frac{1}{\frac{9}{7}}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$$

Answer 39.

$$\sec A = \frac{17}{8}$$

$$\Rightarrow \cos A = \frac{8}{17} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\text{Perpendicular} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$

$$= \sqrt{(17)^2 - (8)^2} = \sqrt{289 - 64} = \sqrt{225} = 15$$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{15}{8}$$

$$\text{To prove}: \frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$$

$$\text{L.H.S} = \frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - 4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2 - 3} = \frac{3 - \frac{900}{289}}{\frac{256}{289} - 3} = \frac{\frac{867 - 900}{289}}{\frac{256 - 867}{289}} = \frac{-33}{-611} = \frac{33}{611}$$

$$\text{R.H.S} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A} = \frac{3 - \left(\frac{15}{8}\right)^2}{1 - 3\left(\frac{15}{8}\right)^2} = \frac{3 - \frac{225}{64}}{1 - \frac{675}{64}} = \frac{\frac{192 - 225}{64}}{\frac{64 - 675}{64}} = \frac{-33}{-611} = \frac{33}{611}$$

$$\Rightarrow \text{L.H.S} = \text{R.H.S}$$

Answer 40.

$$3\tan\theta = 4$$

$$\Rightarrow \tan\theta = \frac{4}{3} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$

$$= \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{5}{3}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{5}{4}$$

$$\text{To prove}: \frac{\sqrt{\sec\theta - \text{cosec}\,\theta}}{\sqrt{\sec\theta + \text{cosec}\,\theta}} = \frac{1}{\sqrt{7}}.$$

$$\frac{\sqrt{\sec\theta - \text{cosec}\,\theta}}{\sqrt{\sec\theta + \text{cosec}\,\theta}} = \frac{\sqrt{\frac{5}{3} - \frac{5}{4}}}{\sqrt{\frac{5}{3} + \frac{5}{4}}} = \frac{\sqrt{\frac{20 - 15}{12}}}{\sqrt{\frac{20 + 15}{12}}} = \frac{\sqrt{\frac{5}{12}}}{\sqrt{\frac{35}{12}}} = \frac{\sqrt{5}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{35}} = \frac{\sqrt{5}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{5} \times \sqrt{7}}$$

$$= \frac{1}{\sqrt{7}}$$

Answer 41.

$$\tan\theta = \frac{m}{n} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\text{Hypotenuse} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$

$$= \sqrt{m^2 + n^2}$$

$$\sin\theta = \left(\frac{m}{\sqrt{m^2 + n^2}}\right)$$

$$\cos\theta = \left(\frac{n}{\sqrt{m^2 + n^2}}\right)$$

To show: $\dfrac{m\sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta} = \dfrac{m^2 - n^2}{m^2 + n^2}$.

$$\frac{m\sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta} = \frac{m\left(\frac{m}{\sqrt{m^2+n^2}}\right) - n\left(\frac{n}{\sqrt{m^2+n^2}}\right)}{m\left(\frac{m}{\sqrt{m^2+n^2}}\right) + n\left(\frac{n}{\sqrt{m^2+n^2}}\right)}$$

$$= \frac{\frac{m^2 - n^2}{\sqrt{m^2+n^2}}}{\frac{m^2+n^2}{\sqrt{m^2+n^2}}} = \frac{m^2 - n^2}{\sqrt{m^2+n^2}} \times \frac{\sqrt{m^2+n^2}}{m^2+n^2}$$

$$= \frac{m^2 - n^2}{m^2 + n^2}$$